39- Mavzu: Kvadrikaning markazi va tasnifi .Uch ulchovli Yevklid fazosidagi kvadrikalar.

Darsning rejasi va maqsadi

1. Kvadrikaning markazi.
2. Kvadrikaning tasnifi.

- **Maqsadi :** Kvadrika tushunchasi, Kvadrikaning markazi, Kvadrikaning tasnifi, Uch ulchovli Yevkiled fazosidagi kvadrikalar xaqida tasavvurlar hosil qilish.

Kesmaning urta nuktasi affin almashtirishda shu kesma obrazi ning urta nuktasiga utadi, shunga asoslanib A_n da kvadrikaning simmetriya markazi tushunchasini kiritish mumkin.

T a ‘ r i f. Kvadrikaning xar bir nuktasiga uning biror S nuktaga nisbatan simmetrnk nuktasi mavjud bulsa, S nukta kvadrikaning simmetriya markazi deb ataladi.

Masalan, A_3 dagi reierda kanonik tenglamasi bilan berilgan ellipsoid, bir va ikki pallaln giprboloidlar uchun koordinatalar boshi simmetriya markazidir.

Kvadrika (1) tenglama bilan berilsa, uning simmetriya markazi koordinatalar boshida bulsa, uning tenglamasi shu reierda (1) lar boshidan iborat va, aksincha, kvadrikaning markazi koor-kurinishda buladn. Xaqiqatan xam, M(u_1, u_{2},... u_n)€(1)=> => M($-u_1$, $-u_{2}$,... $-u_1$ ... . $-u_n$)€ (1)

M M ' kesmaning urta nuktasi O (0, 0, . . . , 0) dir, chunki kesmaning uchlari uning urta nuktasiga nisbatan simmetrik joylashgan. Bundan, tenglamalari ix —- 0, u_1 = 0, u_2=0 . . . , u_k= 0 dan iborat (n — k) ulchovli tekislikning barcha nu$q$talari (1) tenglama bilan aniklanadigan kvadrikaning simmetriya markazi buladi deb chi$q$aramiz. Xususiy xolda k = n bulsa, simmetriya markazlari tuplami nolь ulchovli tekislik bulib, fakat bitta nuktadan, u xam bulsa, koordinatalar boshidan iborat.

U vaktda kvadrika fakat bitta simmetriya markaziga ega bulib, u markazli kvadrika deb ataladi.

Endi kvadrikaning tenglamasi $\varphi_2 + 2\varphi_1 + \alpha_0 = 0$ (2) kurinishda berilgan bulsa, bu kvadrika markazining mavjudligi masalasiga tuxtalaylik.

Kvadrika $\varphi_2 + \alpha_0 = 0$ (3) ko'rinishdagi (bunda φ_2 ifoda n uzgaruvchili kvadratik forma) tenglama bilan berilsa, uning simmetriya markazi koordinatalar boshidan iborat.

Endi (2) kurinishga mos xolni kuraylik. Faraz $q$ilayli$q$, S ($x_1{}^0$, $x_2{}^0$,…,$x_n{}^0$) nu$q$ta (2) kvadrikaning simmetriya markazi bulsin, Reper boshini shu nuktaga kuchiramiz, bazis vektorlarning yunalishini esa ca$q$lab $q$olamiz :

$x_1 = y_1 + x_1{}^0$, $x_2 = y_2 + x_2{}^0$, … , $x_n = y_n + x_n{}^0$ (4)

Bularni (2) ga $q$uyib, soddalashtirsak,

$\varphi_2{}' + (2a_{11}{}'x_1{}^0 + 2a_{12}{}'x_2{}^0 + \ldots + 2a'_{1n}x_n{}^0 + 2a_1)\ y_1 + \ldots + (2a_{n1}{}'x_1{}^0 + 2a_{n2}{}'x_2{}^0 + \ldots + 2a_{nn}{}'x_n{}^0 + 2a_n)\ y_n$ (5)

bunda $\varphi_2{}'$ ifoda y_1, y_2, . . . , y_n uzgaruvchili kvadratik forma, α' - barcha ozod sonlarning algebraik yigindisi. Koordinatalar boshi kvadrikaning simmetriya markazi bulishi uchun (5) tenglama (3) kurinishni olishi kerak, ya'ni birinchi darajali xadlarning barcha koeffitsientlari bir va$q$tda nolga teng bulishi yetarli va zarurdir:

$a_{11}x_1{}^0 + a_{12}x_2{}^0 + \ldots + a_{1n}x_n{}^0 = -a_1$

$a_{21}x_1{}^0 + a_{22}x_2{}^0 + \ldots + a_{2n}x_n{}^0 = -a_2$ (6)

………..

$a_{n1}x_1{}^0 + a_{n2}x_2{}^0 + \ldots + a_{nn}x_n{}^0 = -a_n$

Demak, kvadrika simmetriya markazining $x_1{}^0$, $x_2{}^0$,…,$x_n{}^0$ koordinatalari (6) ni $q$anoatlantirishi kerak, demak, kvadrika markazining mavjudligi masalasi (6) sistemaning yechimiga bogli$q$; $q$uyidagi determinantni $q$araylik:

$$\begin{vmatrix} a_{11} & a_{12} & \cdots a_{1n} \\ a_{21} & a_{22} & \ldots a_{2n} \\ & \ldots & \\ a_{n1} & a_{n2} & \ldots a_{nn} \end{vmatrix} = \Delta$$

1. $\Delta \neq 0$ (6) sistema yagona yechimga ega, kvadrika bitta simmetriya markaziga ega; markazli deb atalgan kvadrika xosil $q$ilinadi.

2. $\Delta = 0$ va (6) sistema cheksiz kup yechimga ega bulsa, kvadri

kaning simmetriya markazlari xam cheksiz kup buladi (bunday nuktalar tuplami k ulchovli tekislik buladi).
3. $\Delta = 0$ va (6) sistema birgalikda bulmasa, kvadrika bit ta xam simmetriya markaziga ega emas. Keyingi ikki xolda kvadrika markazsiz deb ataladi.
E s l a t m a . (6) sistemaning birinchi tenglamasiga dikkat bi lan $q$arasak,u Q: $a_{11}\,x_1\,x_1 + 2a_{12}\,x_1\,x_2 + \ldots + a_{1n}\,x_1\,x_n + a_{22}\,x_2\,x_2 + 2a_{23}\,x_2\,x_3 + \ldots + a_{nn}\,x_n\,x_n + 2a_1\,x_1 + 2a_2\,x_2 + \ldots + 2a_{nn}\,x_n + \alpha_0$ (7) tenglamadan x_1 buyicha ($q$olgan x_2 , x_3, … , x_n larni doimiy deb olinsa) olingan xosiladan, ikkinchi tenglama esa (7) dan x_2 buyicha olingan.

Kvadrikaning tasnifi

n ulchovli affin fazodagi kvadrikaning (7) kurinishdagi

tenglamasini affin reperni maxsus tanlab olish yuli bilan

I. $\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+ \ldots+\varepsilon_k u_k{}^2=1$,k$\leq$ 1, $\varepsilon_i=\pm 1$

II. $\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+ \ldots+\varepsilon_k u_k{}^2=0$,k$\leq$ 1, $\varepsilon_i=\pm 1$ (8)

III. . $\varepsilon_1 u_1{}^2+.\ \varepsilon_1 u_1{}^2 + \varepsilon_2 u_2{}^2 + \ldots+\varepsilon_k u_k{}^2 = 2\varepsilon_{k+1}$, k $\leq 1,\ \varepsilon_i = \pm 1$

kurinishdagi uchta tenglamaning biriga keltirish mumkin. Xech $q$anday affin almashtirish bilan bu tenglamalardan birini ikkinchisiga utkazib bul maydi, demak, ular uzaro affin ekvivalent sinflar emas.

SH u tenglamalarning xar birini ayrim-ayrim kurib chikaylik

1. $\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+ \ldots+.\ \varepsilon_k u_k{}^2 = 1$=1 ,k$\leq$ 1, k=n da

. $\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+ \ldots+ \varepsilon_n u_n{}^2=1$ (9)

1- x o l. $\varepsilon_1=\varepsilon_2=\ldots=\varepsilon_n=1$ uchun
$u_1{}^2+u_2{}^2+ \ldots+u_n{}^2=1$ (10)
xosil $q$ilinib, kvadrika ellipsoid deb ataladi (n= 3 da A_3 dagi ellipsoid).

2- x o l . $\varepsilon_1=\varepsilon_2=\ldots=\varepsilon_n = -1$; A_n bulsa, (9) => da bu tenglamani $q$anoatlantiruvchi birorta xa$qiqiq$iy nu$q$ta yu$q$, bu xolda (10) tenglama mavxum ellipsoidni ani$q$laydi deymiz.

3- xol. ε_1=ε_2=…=ε_n=1 , ε_{i+1}=ε_{i+2}=...= ε_n=-1; bu xolda (9) tenglama bilan ani$q$lanadigan kvadrika n — t indeksli giperboloid deb ataladi (n= 2 x,ol yuz bersa, ε_1 = 1, ε_2 = — 1 yoki ε_1= —1, ε_2 = 1 da kvadrika tekislikdagi giperbolani ifoda $q$iladi, n = 3 da ye,, ye2, ye3 dan bittasi — 1 ga teng bulsa, kvadrika bir pallali giperboloidы i, ε_1, ε_2, ε_3 dan ikkitasi — 1 ga teng bulsa, kvadrika ikki pallali giperboloidni ani$q$laydi). Endi k < n bulgan xolni kuraylik.

$$\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+ \quad ...+ \quad \varepsilon_k u_k{}^2=1. \tag{11}$$

Ma’lumki, bu kurinishdagi tenglama simmetriya markazlari (n — 1) ulchovli koordinata tekisligidan iborat bulgan sirtni ifoda $q$iladi, bunday kvadrika da tsilindrik sirt deb ataladi.

1-xo l . ε_1=ε_2=…=ε_k=1 (11) tenglama $u_1{}^2$+$u_2{}^2$+ ...+$u_n{}^2$=1 (12) ko’rinishni oladi va k ulchovli tekislikdagi ellipsoidni aniklab, A_n fazoda esa asosi shu ellipsoiddan, yasovchilari

(n — k) ulchovli tekislikdan iborat elliptik tsilindrni beradi. n = 3, k = 2 da esa A_3 da yasovchьlari biror koordinata u$q$iga parallel elliptik tsilindrni ani$q$laydi.

2-x o l; ε_1=ε_2 =…=ε_k=-1 uchun $u_1{}^2$+$u_2{}^2$+ ...+$u_n{}^2$=-1; bu tenglama birorta xam xaqiqiy nu$q$taga ega bulmagan kvadrikani anilab, uni mavxum tsilindr deyiladi.

3- xol. ε_1=ε_2 =…=ε_k=1 , ε_{i+1}=ε_{i+2}=...= ε_n=-1 uchun (13)

$$u_1{}^2+u_2{}^2+ \quad ...u_t{}^2-u_{t+1}{}^2- \; ...-u_k{}^2=1$$

Bu tenglama k ulchovli tekislikda (k — t) indeksli giperboloidni aniklab, uning x>ar bir nu^tasidan (n — k) ulchovli tekislik utadi.

Bunday kvadrikani Ap da (k — t) indeksli giperbolik tsilindr deb ataladi; uning yasovchilari (n — k) ulchovli tekislikdan iborat

II. $\varepsilon_1 u_1{}^2+\varepsilon_2 u_2{}^2+$ ...+ $\varepsilon_k u_k{}^2$=0.

Bu tenglama bilan aniklangan kvadrikaning simmetriya markazi koordinatalar boshida bulib, bu nukta kvadrikaga tegishlidir.

k =n bulsin.

1- x ol ε_1=ε_2=…=ε_n bulsa, (14) =► $u_1{}^2$+$u_2{}^2$+ ...+$u_n{}^2$=0

tenglama bilan aniklanadigan kvadrika mavxum konus deb ataladi, bu konus fakat bitta xakikiy nuktaga ega buladi (koordinatalar boshi O).
2- xol. $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ ning barchasi bir xil ishorali bulmasa, kvadrika konus deb ataladi, demak, konus markazli sirtdir.Uning markazi konusning uchi deb ataladi. SHunisi izikki, bu konusga tegishli biror T nuktani olsak, OT tugri chizikning (O — konusning markazi) barcha nuktalari xam konusga tegishli buladi; bu tugri chizik konusning yasovchisi deb ataladi.
Endi k < n xolni tekshiraylik.

1- xo l . ε_1=ε_2=…=ε_k (14) tenglama

$u_1{}^2$+ $u_2{}^2$ ...+$u_n{}^2 = 0$ (15)
kurinishni oladi; bu tenglama bilan aniklanadigan kvadrika xam mavxum konus deb yuritiladi. Lekin bu tenglamani A_n da karasak, bu kvadrika (n — k) ulchovli tekislikning barcha nuktalarini uz ichiga oladi

(chunki N (0, 0,. ., 0, u_{k+1} , . . . , u_n) kurinishdagi barcha nuktalarning koordinatalari (15) tenglamani kanoatlantiradi). Bunday konus uchi (n — k) ulchovli tekislikdan iborat mavxum konus deb ataladi.
2- x o l . ε_1, ε_2,…,ε_k ning barchasi bir xil ishorali bulmasa (masalan, t tasi +1 bulsa), u xolda (14) tenglama bilan anik-lanadigan kvadrikani (k — t) indeksli, uchi (n — k) ulchovli tekislikdan iborat konus deb ataladi. Nixoyat, (8) dagi uchinchi tenglamani tekshiraylik:

$\varepsilon_1 u_1{}^2$+$\varepsilon_2 u_2{}^2$+ ...+ $\varepsilon_k u_k{}^2$=2 u_{k+1} (16)
k = n — 1. 1- xo l . ε_1=ε_2=…=ε_{k-1}; (16) tenglama bilan aniklanadigan kvadrika elliptik paraboloid deb ataladi

(n=3 bulsa, (16) tenglama $u_1{}^2$+$u_2{}^2$= 2 u_3 kurinishda bulib, A_n dagi elliptik paraboloidni ifodalaydi).
2- xol . ε_1, ε_2,…,ε_{n-1} ning barchasi bir xil ishorali bul-masa (masalan, t tasi +1 bulsa), u xolda (16) tenglama bilan aniklanadigan kvadrika (k — t) indeksli giperbolik paraboloid deb ataladi.

k ≤ n— 2. U xolda (16) tenglama O nukta va

$\overrightarrow{e_1}$, $\overrightarrow{e_2}$, . . . , $\overrightarrow{e_{r+1}}$
vektorlar bilan aniklanadigan tekislikda biror paraboloid ni aniklaydi. A_n da karasak, bu kvadrikaga (n — k — 1) ulchovli tekislik kiradi, anikrogi N nukta paraboloidga tegishli bulsa, u xolda boshlari shu nuktadagi $\overrightarrow{e_{k+2}}$, … $\overrightarrow{e_n}$, vektorlar bilan aniklanuvchi tekislik shu paraboloid tarkibida buladi.

Bu xolda (16) kvadrika yasovchilari (k — 1) ulchovli tekislikdan iborat parabolik tsilindr deb ataladi. Bu kvadrikaning indeksi (n— t) bulsa, u mos ravishda (n — t) indeksli parabolik tsilindr deb ataladi.
M i s o l: A_3 $u_1{}^2$+ $u_2{}^2$+4 $u_1 u_2$-4u_2 da tenglama bilan aniklanuvchi kvadrikaning turini toping.

Uch ulchovli yevklid fazosidagi kvadrikalar

p ulchovli affin fazodagi kvadrikalar tasnifi bilan mufassil tanishdik. Uch ulchovli affin fazoda 17 xil kvadrikaning borligini oshkor kilish osondir. (8) dagi tenglamalarda k ni 1, 2, 3 sonlar deb olinsa 17 ta xar xil tenglama xosil kilamiz. SH u kvadrikalarni uch ulchovli yevklid fazosida karasak, dekart reperini kulay tanlab olish yuli bilan ;ularning tenglamalarini kuyidagi jadvalda kursatilgandek kilib yozish mumkin (uzgaruvchilarni ii i2, щ bilan emas, balki eskicha belgilashimizga mos ravishda x, u, z deb olamiz

№	Квadrikaning sodda tenglamasi	Kvadrikaning nomi
1	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	ELLIPSOID
2	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$	mavxum ellipsoid
3	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	bir pallali giperboloid
4	$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	ikki pallali giperboloid
5	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$	mavxum konus
6	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	uchi koordinatalar boshida bulgan konus

7	$\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1	elliptik tsilindr
8	$\frac{x^2}{a^2}+\frac{y^2}{b^2}$ =-1	mavxum tsilindr
9	$\frac{x^2}{a^2}$ - $\frac{y^2}{b^2}$=1	mavxum tsilindr
10	$\frac{x^2}{a^2}+\frac{y^2}{b^2}$=0	Oz uk buyicha kesishuvchi 2 ta mavxum tekisl

№	Kvadrikaning sodda tenglamasi	Kvadrikaning nomi
11	$\frac{x^2}{a^2}$ - $\frac{y^2}{b^2}$ =0	ikkita kesishuvchi tekislik
12	$\frac{x^2}{a^2}$ =1	ikki uzaro parallel tekislik
13	$\frac{x^2}{a^2}$ =-1	ikki mavxum uzaro parallel te kislik
14	x^2=0	ikki mavxum uzaro parallel te kislik
15	$\frac{x^2}{a^2}+\frac{y^2}{b^2}$ =2z	ikki mavxum uzaro parallel te kislik

16	$\frac{x^2}{a^2}$ - $\frac{y^2}{b^2}$ =2z	ikki mavxum uzaro parallel te kislik
17	$\frac{x^2}{a^2}$=2z	ikki mavxum uzaro parallel te kislik

Konus kesmlari ikkinchi tartibli chiziqlar bo'lib ularni umumiy tenglamasini quyidagi kvadratik forma yordamida ifodalasa bo'ladi.

$$\begin{aligned} F(x_1,x_2) &= A_1{x_1}^2+2A_4x_1x_2+A_2{x_2}^2+2A_5x_1+2A_6x_2+A_3 = \\ &= (x_1,x_2,1)\begin{pmatrix} A_1 & A_4 & A_5 \\ A_4 & A_2 & A_6 \\ A_5 & A_6 & A_3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}, \end{aligned} \qquad (8)$$

Bunda ikkinchi tartibli chiziqning turi $\Delta = ({A_4}^2 - A_1A_2)$. diskremenantning ishorasi bilan aniqlanadi.

Yuqoridagi kvadrikaning koffetsientlaridan tuzilgan matritsani A bilan belgilaymiz va uni determinanti quyidagicha aniqlanadi

$$detA = A_3(A_1A_2 - {A_4}^2) - A_1{A_6}^2 - A_2{A_5}^2 + 2A_4A_5A_6\ . \qquad (9)$$

Ikkinchi tartibli chiziq uchun F n – darajali kupxad uchun o'rinli bo'lgan

$$F(\mathbf{x}) = 0 \Rightarrow \nabla F \cdot \mathbf{x} = \quad \nabla F \cdot \mathbf{x} - nF$$ ayniyatdan foydalanamiz

u xolda quyidagilarga ega bo'lamiz[1]

$$\begin{aligned} x &= \frac{A_4x_1 + A_2x_2 + A_6}{[(A_1+A_4)x_1 + (A_2+A_4)x_2 + (A_5+A_6)]} \\ y &= -\frac{A_5x_1 + A_6x_2 + A_3}{[(A_1+A_4)x_1 + (A_2+A_4)x_2 + (A_5+A_6)]}\ . \end{aligned} \qquad (10)$$

[1] PARALLEL COORDINATES : VISUAL Multidimensional Geometry and its Applications Alfred Inselberg, mazmun – mohiyatidan foydalanildi
(2004)

Konus kesmlarning klassifikatsiyasi

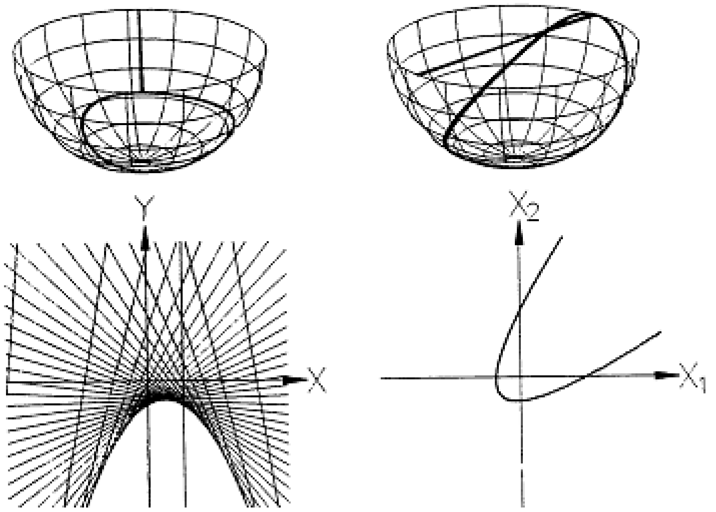

Figure 19: A parabola whose ideal point has direction with slope 1 transforms to a parabola - self-dual.

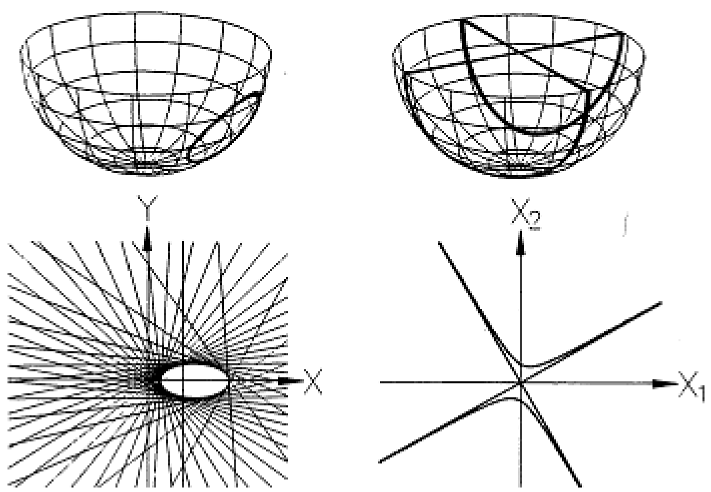

Figure 20: Hyperbola to ellipse – dual of case shown in Fig. 17

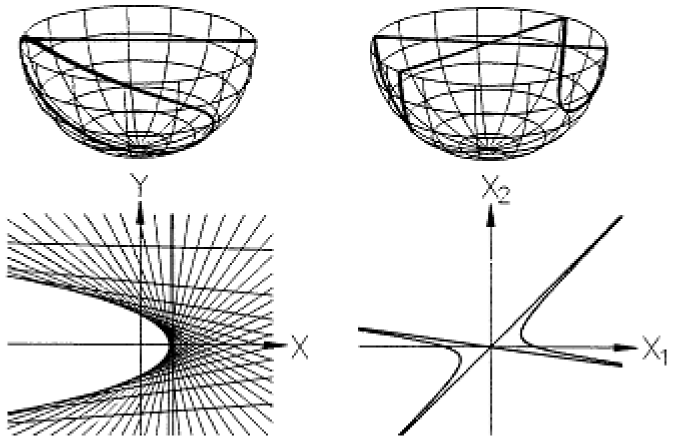

Figure 21: Hyperbola to parabola. This occurs when one of the assymptotes has slope 1 – dual of case shown in Fig. 18

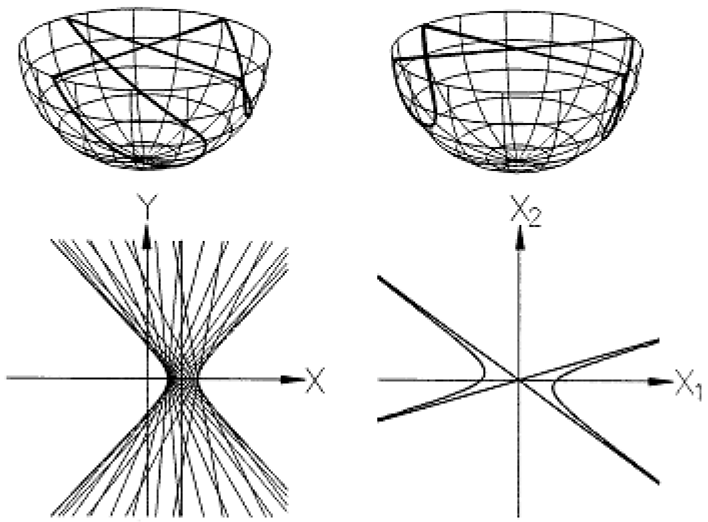

Figure 22: Hyperbola to hyperpola – self-dual case.