Chiziqliakslantirishlarvachiziqlioperatorlar.

Reja:

- Chiziqli akslantirishlar.
- Chiziqlioperatorlar.

Biz oldingima'ruzalardavektorfazotushunchasibilantanishganedik.Endituрlivektorlarfazolariorasidaqandaymunosabatlarmavjudliginiko'raylik.

U vektorfazoning V vektorfazogaakslantirish φ bo'lsa, u holda $\varphi: U \to V$ ko'rinishdabelgilaylik.U vektorfazoningixtiyoriy $\bar{x}$ elementiga φ akslantirishyordamida V vektorfazodanmoskeluvchivektorni $\bar{y}$ deylik. Bu moslik $\varphi: \bar{x} \to \bar{y}$, $\bar{x} \xrightarrow{\varphi} \bar{y}$, $\varphi\bar{x} = \bar{y}$, $y = \varphi(\bar{x})$ ko'rinishlardabelgilanadi.

23.1-Ta'rif.$\mathcal{F}$sonlarmaydoniustidaaniqlangan U vektorfazoning V vektorfazogaakslantiruvchi φ akslantirishuchunushbu

1. $\varphi(\bar{x}_1 + \bar{x}_2) = \varphi(\bar{x}_1) + \varphi(\bar{x}_2)$,

2. $\varphi(\lambda\bar{x}) = \lambda\varphi(\bar{x}) \quad (\lambda \in F)$

shartlarbajarilsa, u holda U vektorfazo V vektorfazogachiziqliakslanadideyiladi.[1]

[1]Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.

5.1.1. Definition. *Let A and V be vector spaces over the same field F. The mapping $f : A \longrightarrow V$ is called a linear mapping, or a homomorphism of vector spaces, if it satisfies the following properties:*

$$f(x+y) = f(x) + f(y) \text{ and } f(\alpha x) = \alpha f(x)$$

for all $x, y \in A, \alpha \in F$. An injective linear mapping is called a monomorphism, a surjective linear mapping is called an epimorphism, and a bijective linear mapping is called an isomorphism.

U fazoni V fazogachiziqliakslantirishlarto'plamini $Hom(U,V)$ orqalibelgilanadi.

23.2-Ta'rif. U vektorfazonio'z-o'zigaakslantirish U fazodaaniqlangan operator deyiladi.

Yuqoridagiikkitata'rifdanko'rinadiki, operator chiziqliakslantirishningxususiyholiekanligi.

Operatorlar $f, \varphi, ...$ harflarbilanbelgilanadi.

23.3-Ta'rif. U vektorfazonio'z-o'zigachiziqliakslantirish U fazodaaniqlanganchiziqli operator deyiladi.

φ chiziqliakslantirishta'sirida $\varphi(\overline{x}) = \overline{y}$ bo'lsa, u holda $\overline{y}$ vektor $\overline{x}$ vektorningobrazi (tasviri), $\overline{x}$ vektoresa $\overline{y}$ vektorningproobrazi (asli) deb yuritiladi.

$\overline{x} \in U$ bo'lganda $\varphi(\overline{x}) \in V$ vektorlarto'plamiodatda φ akslantirishningobrazi deb yuritiladiva $Jm\varphi$ yoki φU orqalibelgilanadi.

23.4-Misol. Agar $\varphi : \alpha \to \overline{\alpha}$ akslantirish S kompleks sonlarmaydoniustidachiziqli operator bo'ladi (Bunda $\overline{\alpha}$ va α sonlaro'zaroqo'shmakomplekssonlar).

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.

23.5-Ta'rif. U vektorfazoningixtiyoriy $\bar{x}_1$ va $\bar{x}_2$ elementlariva U da aniqlangan φ operator uchun $\varphi(\bar{x}_1+\bar{x}_2)=\varphi(\bar{x}_1)+\varphi(\bar{x}_2)$ tenglikbajarilsa, u holda φ ga U da aniqlanganadditiv operator deyiladi.

Quyidagixossalaro'rinli:

1^0. $\varphi 0 = 0_1$;

2^0. $\varphi(-\bar{x}) = -\varphi(\bar{x}) \quad (\forall \bar{x} \in U)$;

3^0. $\varphi(r\bar{x}) = r\varphi\bar{x} \quad (\forall r \in Q)$;

4^0. $\varphi(\bar{x}_1-\bar{x}_2)=\varphi(\bar{x}_1)-\varphi(\bar{x}_2) \quad (\forall \bar{x}_1, \bar{x}_2 \in U)$.

5.1.3. Proposition. *Let A, V be vector spaces over a field F and let $f : A \longrightarrow V$ be a linear mapping. Then the following properties hold:*

(i) $f(0_A) = 0_V$.

(ii) $f(-x) = -f(x)$ *for all elements* $x \in A$.

(iii) $f(x-y) = f(x) - f(y)$ *for all* $x, y \in A$.

(iv) $f(\alpha_1 x_1 + \cdots + \alpha_n x_n) = \alpha_1 f(x_1) + \cdots + \alpha_n f(x_n)$ *for all* $x_1, \ldots, x_n \in A$ *and* $\alpha_1, \ldots \alpha_n \in F$.

(v) *If B is a subspace of A, then its image $f(B)$ is a subspace of V; in particular,* $f(A) = \mathbf{Im} f$ *is a subspace of V.*

(vi) *If U is a subspace of V, then its preimage $f^{-1}(U)$ is a subspace of A; in particular,*

$$\mathbf{Ker} f = \{x \in A \mid f(x) = 0_V\} = f^{-1}(\{0_V\})$$

is a subspace of A.

(vii) *If M is a subset of A, then* $\mathbf{Le}(f(M)) = f(\mathbf{Le}(M))$.

23.6-Ta'rif. Agar λ ixtiyoriy son bo'lganda ham U fazoningixtiyoriy $\bar{x}$ elementiuchun $\varphi(\lambda\bar{x})=\lambda\varphi(\bar{x})$ tengliko'rinlibo'lsa, u holda φ ga U da aniqlanganbirjinsli operator deyiladi.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.

23.7-Ta’rif. Birvaqtdabirjinslivaadditivbo’lganoperatorgachiziqli operator deyiladi.

φ operatorchiziqli operator bo’lishiuchun U fazoningixtiriy $\overline{x}_1$ ва $\overline{x}_2$ elementlariva $\lambda_1, \lambda_2 \in F$ berilganda $\varphi(\lambda_1 \overline{x}_1 + \lambda_2 \overline{x}_2) = \lambda_1 \varphi(\overline{x}_1) + \lambda_2 \varphi(\overline{x}_2)$ tenglikningbajarilishizarurvaetarli.

Bu mulohazaniisbotlashdayuqoridagiikkitata’rifdanfoydalaniladi.

Agar φ chiziqlioperatorbo’lsa, u holda $\forall x_i \in U, \quad \lambda_i \in P \ (i = \overline{1,n})$ uchunushbu

$$\varphi(\lambda_1 \overline{x}_1 + \lambda_2 \overline{x}_2 + ... + \lambda_n \overline{x}_n) = \lambda_1 \varphi(\overline{x}_1) + \lambda_1 \varphi(\overline{x}_2) + ... + \lambda_n \varphi(\overline{x}_n) \qquad (1)$$

tengliko’rinlibo’ladi.

Bumulohazada (1) tenglikmatematikinduktsiyaprintsipiasosidaisbotqilinadi.

23.8-Ta’rif. Agar $\forall \overline{x} \in U$ uchun $\varphi(\overline{x}) = 0$ tenglikbajarilsa, u holda φ operatorganoloperatordeyiladi.

Noloperatorhamchiziqlioperatorbo’ladi. (Isbotlang).

23.9-Ta’rif. Agar $\forall \overline{x} \in U$ uchun $e(\overline{x}) = \overline{x}$ tenglikbajarilsa, u holda e gaayniy (birlik) operatordeyiladi.

23.10-Ta’rif. Agar $\forall \overline{x} \in U, \quad \lambda \in P$ uchun $\varphi(\overline{x}) = \lambda \overline{x}$ tenglikbajarilsa, u holda φ gao’xshashlikoperatorideyiladi.

Demak, buta’rifdanko’rinadiki, $\lambda = 0$ bo’lsa, o’xshashlikoperatoriningnoloperator, $\lambda = 1$ bo’lsa, o’xshashlikoperatoriningayniyoperatorbo’lishi.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, “ALGEBRA AND NUMBER THEORY” pp.187-200.

23.11-Ta'rif. Agar $\bar{x}=(x_1,x_2,...,x_n)\in U$ bo'lib, $\varphi(\bar{x})=\varphi(x_1,x_2,...,x_n)=(x_1,x_2,...,x_k)\quad(1\le k<n)$ bo'lsa, ya'ni φ operator n o'lchovlifazodagivektorni k o'lchovlifazodagivektorgao'tkazuvchioperatorbo'lsa, u holda φ gaproektsiyalovchioperatordeyiladi.

23.12-Ta'rif. Agar $U_n$fazoningixtiyoriy $\bar{x}$ vektoriuchun $f(\bar{x})=\varphi(\bar{x})+\Psi(\bar{x})$ tenglikbajarilsa u holda f ga φ va Ψ operatorlarningyig'indisideyiladiva u $\varphi+\Psi=f$ orqaliyoziladi.

23.13-Ta'rif. $\alpha\in F$, $\forall\bar{x}\in U_n$ uchun $(\alpha\varphi)\bar{x}=\alpha\varphi(\bar{x})$ tenglikbajarilsa, u holda $\alpha\varphi$ ga φ operatorning α skalyargako'paytmasideyiladi.

AyrimhollardaU$_n$fazoningnolmasvektorini φ operatorta'siridanolvektorgaakslanishimumkin.

Takrorlashuchunsavollar:

1. Chiziqliakslantirish deb nimagaaytiladi.?
2. Chiziqli operator deb nimagaaytiladi?
3. Additiv operator deb nimagaaytiladi?
4. Birjinsli operator deb nimagaaytiladi?
5. Nol operator deb nimagaaytiladi?
6. Birlik operator deb nimagaaytiladi?
7. O'xshashlik, proektsiyalovchioperatorlar deb nimagaaytiladi?
8. Chiziqlioperatorlarustidaqandayamalarnibilasiz?

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.

Foydalaniladigan adabiyotlar ro'yxati

Asosiy adabiyotlar:

1. MalikD.S., MordesonJ.N., SenM.K. Fundamentalofabstractalgebra. WCB McGrew-Hill, 1997.
2. Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" 2010.
3. Кострикин А.М. Введение в алгебру.- М.- «Мир».- 1977.
4. Под ред. Кострикина, Сборник задач по алгебре, М.Наука, 1986.
5. Хожиев Ж.Х. Файнлейб А.С. Алгебра ва сонлар назарияси курси, Тошкент, «Ўзбекистон», 2001 й.
6. Курош А.Г. Олий алгебра курси, Тошкент, «Ўқитувчи». 1975й.
7. Гельфанд И.М. Чизиқлиалгебраданлекциялар. «Олийваўртамактаб». 1964.
8. Р.Н.Назаров, Б.Т. Тошпўлатов, А.Д.Дусумбетов, Алгебра ва сонлар назарияси 1 қисм, 2 қисм, 1993й., 1995й.
9. A.Yunusov , D.Yunusova , Algebra va sonlar nazariyasi. Modultexnologiyasiasosidatuzilganmusolvamashqlarto'plami. O'quvqo'llanma. 2009.

Qo'shimcha adabiyotlar:

1. Фаддеев Д.К. Лекции по алгебре, М., "Наука"1984г.
2. Фаддеев Д.К., Соминский И.С. Сборник задач по высшей алгебре, М.: Наука, 1977 г.
3. Поскуряков И.Л. Сборник задач по линейной алгебре. «Наука», 1978г.

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.

4. Ламбек И. Кольца и модули.- М.- «Мир».- 1971.

5. Херстейн. Некоммутативные кольца. М.- «Мир».- 1967.

6. VilnisDetlovs,KarlisPodnieks,Introduction to MathematicalLogic. University of Latvia. Version released: August 25, 2014.

7. А.Юнусов , Д.Юнусова, М.Маматқулова, Г.Артикова, Модул технологияси асосида тайёрланган мустақил ишлар тўплами. 1−3−қисмлар, 2010.

8. Скорняков Л.Ф. Элементи обшей алгебри. М., 1983 г.

9. Петрова В.Т. лексия по алгебре и геометрии. Ч.1,2. Москва,1999г.

10. YunusovA.S. Matematikmantiqvaalgoritmlarnazariyasielementlari. T., “Yangiasravlodi”. 2006.

11. YunusovA., YunusovaD. Sonlisistemalar. T., «Moliya−iqtisod», 2008.

12. Мазуров В.Д. и др. Краткий конспект курса висшейалгебри.

Elektron ta’lim resurslari

1. www.Ziyo.Net
2. http://vilenin.narod.ru/Mm/Books/
3. http://www.allmath.ru/
4. http://www.pedagog.uz/
5. http://www.ziyonet.uz/
6. http://window.edu.ru/window/
7. http://lib.mexmat.ru;

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, “ALGEBRA AND NUMBER THEORY” pp.187-200.

8. http://www.mcce.ru,
9. http://lib.mexmat.ru
10. http://techlibrary.ru;

*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, "ALGEBRA AND NUMBER THEORY" pp.187-200.