Matritsa vauningrangi.Matritsaningustunva

qator (satr) ranglariningtengligi

Reja:

- Matritsa.
- Nomdoshmatritsalar.
- Matritsalartengligi.
- Matritsaningsatr (ustun) vektorlarisistemasi.
- Matritsaningsatr (ustun) rangi.
- Matritsanielementaralmashtirishlar.
- Pog'onasimonmatritsa.
- Transponirlanganmatritsa.
- Matritsaningsatrvaustunranglariningtengligi.

$F=<F;+,\cdot,-,^{-1},0,1>$ maydonberilganbo'lsin.

20.1-ta'rif. *F*maydonningmntaa$_{ij}$(i=$\overline{1,m}$,j=$\overline{1,n}$) elementlaridan

tuzilganushbu $A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m3} & \dots & a_{mn} \end{pmatrix}$

ko'rinishdagijadval F maydonustidagi $m\times n$ tartiblimatritsadeyiladi.

MatritsaA,B,S,... harflar orqali belgilanadi. a$_{ij}$ lar matritsaning elementlari deyiladi. a$_{ij}$ element, matritsaningi-satri, j-ustuni kesishmasidagi element.

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Matritsada m>n, m<n, m=n bo'lishimumkin. Agar matritsada m=n bo'lsa, u holda bunday matritsa n-tartiblikvadrat matritsa deyiladi.

20.2-ta'rif.A va B matritsalar berilgan bo'lib, ularning mos ravishda satrlari vaustunlari soni teng bo'lsa, u holdaA va B matritsalarni nomdosh matritsalar deb yuritiladi.

Masalan, $\begin{pmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 2 & 4 & 8 & 1 \\ 3 & 6 & 3 & 7 & 3 \\ 1 & 5 & -1 & 3 & 7 \end{pmatrix}$ va $\begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 2 & 3 & 1 & 3 & 8 \\ 2 & 4 & 7 & 22 & 6 \\ 6 & -3 & 5 & 0 & 1 \end{pmatrix}$ matritsalar 4×5 tartibli matritsalar, ya'niular nomdosh matritsalar.

20.3-ta'rif. A matritsaning har bira_{ij} elementi B matritsaningunga mos b_{ij} elementiga teng bo'lsa, u holdaA va B nomdosh matritsalar teng (aks holda teng emas) matritsalar deyiladi.

$A^i = \begin{pmatrix} a_{i1} \\ a_{i2} \\ \vdots \\ a_{in} \end{pmatrix}$ matritsaga n ta satrli, 1 taustunli, $A_j = \begin{pmatrix} a_{j1} & a_{j2} & \cdots & a_{jn} \end{pmatrix}$ matritsaga 1 ta satrli, n taustunli matritsa deyiladi. Bitta satrli matritsalarni satr vektorlar, bittaustunli matritsalarniustun vektorlar deb qarashmumkin.

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

matritsada $\overline{A}_1,\dots,\overline{A}_m$ satrvektorlarva $\overline{A}^1,\dots,\overline{A}^n$ ustunvektorlarmavjud.

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20.4-ta’rif. Matritsadagisatrvektorlarsistemasiningrangigamatritsaningsatrrangi, ustunvektorlarsistemasiningrangigauningustunrangideyiladi. A matritsaningsatrrangini $r(A)$, ustunrangini $\rho(A)$ko’rinishdabelgilaymiz.[1]

4.3.1. Definition. *Let A be a vector space over a field F and let M be a finite subset of A. Then* $\mathbf{dim}_F(\mathbf{Le}(M))$ *is called the rank of the subset M and is denoted by* $\mathbf{rank}(M)$.

Matritsaranginianiqlashuchunmatritsaustidaelementaralmashtirishlarbajariladi.Ularquyidagilar:

1. Matritsadagiixtiyoriyikkitasatryokiustuno’rinlarinialmashtirish.
2. Matritsadagiixtiyoriysatryokiustunelementlarininoldanfarqlisongako’paytirish.
3. Matritsaningsatryokiustunelementlarininoldanfarqlisongako’paytirib, boshqasatryokiustunningmoselementlarigaqo’shish.
4. Barchaelementlarinollardaniboratbo’lgansatryokiustunnimatritsadanchiqarish.

1. $\begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 2 & -1 & 6 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & -1 \\ 2 & -1 & 6 & 4 \\ 0 & 2 & 4 & 3 \end{pmatrix};$

2. $\begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 2 & -1 & 6 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 0 & 4 & 8 & 6 \end{pmatrix};$

[1]Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya.Subbotin, “ALGEBRA AND NUMBER THEORY” pp.174-182.
*Martyn R. Dixon, Leonid A. Kurdachenko, Igor Ya. Subbotin, “ALGEBRA AND NUMBER THEORY” pp.174-182.

3. $\begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 2 & -1 & 6 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 0 & -5 & 4 & 6 \end{pmatrix}$;

4. $\begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & -1 \\ 0 & 2 & 4 & 3 \end{pmatrix}$.

20.1-teorema. Elementaralmashtirishlarmatritsaranginio'zgartirmaydi.

Matritsanielementaralmashtirishlarnatijasidauningsatr (ustun) vektorlarisistemasidaelementaralmashtirishlarbajariladi.Ma'lumki, vektorlarningcheklisistemasinielementaralmashtirishlarnatijasidaundagichiziqlierkl ivektorlarsonio'zgarmaydi, ya'nivektorlarsistemasiningrangio'zgarmaydi.Shuninguchunmatritsanielementaral mashtirishlarnatijasidauningrangio'zgarmaydi.

20.5-ta'rif. Matritsasatriningboshlovchielementidebuningbirinchi (chapdano'nggaqaraganda) noldanfarqlielementigaaytiladi.

20.6-ta'rif. Matritsapog'onasimondeyiladi, agaruningnolqatorlaribarchanolmasqatorlardankeyinjoylashganva $\alpha_{1k_1}, \alpha_{2k_2}, \ldots, \alpha_{rk_r}$ boshlovchielementlariuchun $k_1 < k_2 < \ldots < k_r$ bo'lsa.

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Masalan, $\begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$ matritsapog'onasimonmatritsaemas.

Chunki, 3-, 4- satrlaridaginoldanfarqli (chapdano'ngga) birinchielementlar 3-ustunda joylashgan. Bu matritsaning 3-satrini (-2) gako'paytirib, 5 gako'paytirilgan 4-satrga qo'shamiz:

$$\begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 0 & -11 & 19 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

4-, 5-satrlarning boshlovchielementlari 4-ustunda bo'lganligiuchunyanaelementaralmashtirishbajaramiz. 4-ustunni 3ga, 5-ustunni 11ga ko'paytirib, ularniqo'shamiz:

$$\begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 0 & -11 & 19 \\ 0 & 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 0 & -11 & 19 \\ 0 & 0 & 0 & 0 & 112 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

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Hosilbo'lganmatritsaning 5-satrini 112ga bo'lamizvauni (-4)gako'paytirib, 6-satrga ko'shamiz:

$$\begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 0 & -11 & 19 \\ 0 & 0 & 0 & 0 & 112 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 & 2 & 3 \\ 0 & 3 & 5 & 0 & 2 \\ 0 & 0 & 5 & 8 & 3 \\ 0 & 0 & 0 & -11 & 19 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Hosilbo'lganoxirgimatritsapog'onasimonmatritsa.

20.2-teorema. Harqanday $m \times n$ tartiblimatritsasatrelementaralmashtirishlarnatijasida $m \times n$ tartiblipog'onasimonmatritsagaekvivalentbo'ladi.

20.1-misol. $\begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 11 \end{pmatrix}$ matritsaningranginitopishuchununing uchtasatrvektorlaridaniboratvektorlarsistemaningranginianiqlaymiz. Nolvektorchiziqlibog'liqbo'lganligivavektorlarsistemasidannolvektornichiqarishun ingranginio'zgartirmaganligiuchun, ikkinchiqatornimatritsadanchiqaramiz. Vektorlarsistemasinielementaralmashtirishnatijasidaberilgansistemagaekvivalentsis temahosilbo'lishinie'tiborgaolsak, berilganmatritsagaekvivalent $\begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 2 & 0 & 0 & 0 & 11 \end{pmatrix}$ matritsahosilbo'ladi. Ustunnolvektorlarnimatritsadanchiqarib $\begin{pmatrix} 1 & 5 \\ 2 & 11 \end{pmatrix}$ matritsagaegabo'lamiz.

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Matritsaranginianiqlashjarayonidaustunvasatrvektorlarsistemasidaelementaralmashtirishlarnibajarishmumkin.Hosilqilinganmatritsadaikkitachiziqlibog’lanmagansatrhamdaustunvektorlarsistemalarikelibchiqdi.Demakberilganmatritsaningsatrrangi $r(A)=2$ vauningustunrangi $\rho(A)=2$.

20.2-misol. $\begin{pmatrix} 3 & 5 & 7 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ ~ (birinchivaikkinchisatrlarniqo’shib, birinchi satro’rnigayozamiz) ~ $\begin{pmatrix} 4 & 8 & 12 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ ~ (birinchisatrelementlari $\frac{1}{4}$ ga ko’paytirilgan) ~ $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$ ~ (birinchivaikkinchisatrlartengbo’lganligiuchunbiriniqoldiramiz) ~ $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$.

Berilganmatritsanisatrvektorlarsistemasinielementaralmashtirishlarnatijasidauningsatrrangi $r(A)=2$ ekanligikelibchiqadi.

20.7-ta’rif. A^tmatritsa A matritsaningtransponirlanganideyiladi, agarA^tmatritsa A matritsasatrlariniustunlarorqaliyozishdanhosilbo’lganbo’lsa, ya’ni

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}; \qquad A^t = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{pmatrix};$$

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20.3-misol. A = $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & -4 & 2 \end{pmatrix}$ matritsaningtransponirlashnatijasida

$A^t = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & -4 \\ 3 & 1 & 2 \end{pmatrix}$ hosilbo'ladi.

20.3-teorema. Matritsaningsatrrangiuningustunrangigateng.

4.3.2. Proposition. *Let F be a field and let A be a vector space over F. Suppose that M is a finite subset of A. Then* $\mathbf{rank}(M)$ *is equal to the number of elements in every maximal linearly independent subset of M.*

Takrorlashuchunsavollar:

1. Matritsadebnimagaaytiladi?
2. Nomdoshmatritsalargata'rifbering.
3. Qandaymatritsalartengdeyiladi?
4. Matritsaningsatr (ustun) vektorlarisistemasinima?
5. Matritsaningsatr (ustun) rangidebnimagaaytiladi?
6. Matritsanielementaralmashtirishlardebqandayalmashtirishlargaaytiladi?
7. Matritsaningsatrvaustunranglarihaqidagiasosiyteoremaniayting.

Foydalaniladigan adabiyotlar ro'yxati

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