O’rniga qo’yishlar va o’rinlashtirishlar

Reja:

- n-darajalio’rnigaqo’yish.
- O’rnigaqo’yishlargruppasi.
- n-darajalisimmetrikgruppa.
- Inversiya.
- Juft, toqo’rnigaqo’yishlar.
- Transpozitsiya.
- O’rnigaqo’yishningishorasi.

Bizga n taelementgaegabo’lgan A to’plamberilganbo’lsin.

To’plamelementlarinishartliravishda 1,2,...,n sonlarorqalibelgilabolamiz, ya’niberilganto’plamni A={1,2,3,...,n} ko’rinishdayozishmumkin.

7.1-ta’rif. A={1,2,3,...,n} to’plamnio’zigabiyektivakslantirishga**n-darajalio’rniga**qo’yishdeyiladi.

A to’plamdaaniqlangan φ o’rnigaqo’yishni

$$\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$$

ko’rinishdabelgilanadi.

These arguments show that in order to study permutations of the set $A = \{a_1, a_2, \dots, a_n\}$, we can study permutations of $\{1, 2, \dots, n\}$ (notice that the two sets have the same number of elements). Earlier we used the notation $\mathbf{S}(A)$ for the set of permutations of A. However, the notation $\mathbf{S}(\{1, 2, \dots, n\})$ is cumbersome so we shall instead use the notation $\mathbf{S}_n$ for the set of all permutations of the

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set $\{1, 2, \ldots, n\}$, which is in accord with standard usage. If $\pi \in \mathbf{S}_n$, then we will say that π is *a permutation of degree n*. Every permutation of degree n can conveniently be written as a matrix consisting of two rows, where the first row has the entries $1, 2, \ldots, n$ and $\pi(m)$ is written in the second row under the entry m in the first row. The permutation π can be written as

$$\begin{pmatrix} 1 & 2 & \ldots & n \\ \pi(1) & \pi(2) & \ldots & \pi(n) \end{pmatrix},$$

which we will call the *tabular form of the permutation*. We note that this is just a notational device; we shall not be adding or multiplying such tabular forms in the manner usually reserved for matrices. Since π is a permutation of the set $\{1, 2, \ldots, n\}$, we see that

$$\{1, 2, \ldots, n\} = \{\pi(1), \pi(2), \ldots, \pi(n)\}.$$

Bundabirinchiqatordagielementlarningjoylashishtartibiahamiyatgaegaemas, lekinikkinchiqatorelementlarinijoylashtirgandaharbir k vaungamos $\varphi(k)$ elementlarningbirustundajoylashishigae'tiborberishkerak.

To justify some of these remarks, let A be a set with n elements, say $A = \{a_1, a_2, \ldots, a_n\}$ and let π denote a permutation of A. For $1 \le j \le n$, let $\pi(a_j) = a_k$, where k is dependent upon j. Then π induces a mapping $\pi_0 : \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, n\}$ defined by

$$\pi_0(j) = k \text{ whenever } \pi(a_j) = a_k.$$

A to'plamningbarchao'rnigao'yishlarto'plamini S_n orqalibelgilaymiz.

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7**.1-misol.** $A=\{1,2\}$ to'plamberilganbo'lsa, u yordamidahosilqilinganikkinchidarajalio'rnigaqo'yishlarquyidagiko'rinishdabo'ladi: $\varphi_0=\begin{pmatrix}1 & 2\\ 1 & 2\end{pmatrix}, \varphi_1=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ va $S_2=\{\varphi_0,\varphi_1\}$.

7.2-ta'rif. Agar φ va ψ o'rnigaqo'yishlarda $i_k=j_k(k=\overline{1,n})$ bo'lsa, u holda φ va ψ o'rnigaqo'yishlaro'zarotengdeyiladi.

Masalan, $\varphi=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix}$, $\psi=\begin{pmatrix}1 & 3 & 2 & 4\\ 2 & 3 & 4 & 1\end{pmatrix}$ o'rnigaqo'yishlaro'zaroteng

.

7.3-ta'rif. φ va ψ o'rnigaqo'yishlarko'paytmasideb φ va ψ akslantirishlarkompozitsiyasi $\varphi\psi(i)=\varphi(\psi(i)), i=1,\ldots,n$ gaaytiladi, ya'ni

$$\varphi\cdot\psi=\varphi\cdot\begin{pmatrix}1 & 2 & \ldots & n\\ \psi(1) & \psi(2) & \ldots & \psi(n)\end{pmatrix}=\begin{pmatrix}\psi(1) & \psi(2) & \ldots & \psi(n)\\ \varphi(\psi(1)) & \varphi(\psi(2)) & \ldots & \varphi(\psi(n))\end{pmatrix}.$$

7.4-ta'rif. A to'plamdanolingan φ o'rnigaqo'yishgateskario'rnigaqo'yishdeb

$$\varphi^{-1}=\begin{pmatrix}\varphi(1) & \varphi(2) & \cdots & \varphi(n)\\ 1 & 2 & \cdots & n\end{pmatrix}=\begin{pmatrix}1 & 2 & \ldots & n\\ \varphi^{-1}(1) & \varphi^{-1}(2) & \ldots & \varphi^{-1}(n)\end{pmatrix}$$

o'rnigaqo'yishgaaytiladi.

7.5-ta'rif. A to'plamningharbirelementinishuelementningo'zigao'tkazuvchi ε akslantirishgaayniyo'rnigaqo'yishdeyiladiva u

$$\varepsilon=\begin{pmatrix}1 & 2 & \cdots & i & \cdots & n\\ 1 & 2 & \cdots & i & \cdots & n\end{pmatrix}$$ ko'rinishdabelgilanadi.

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7.1-teorema. A cheklito’plamningbarchao’rnigaqo’yishlarto’plamimultiplikativgruppabo’ladi.

7.6-ta’rif. $< S_n;\ \cdot,^{-1} >$gruppaga n-darajalisimmetrikgruppadeyiladiva u $S_n$orqalibelgilanadi.

7.7-ta’rif. $\varphi = \begin{pmatrix} 1 & 2 & \ldots & n \\ \varphi(1) & \varphi(2) & \ldots & \varphi(n) \end{pmatrix}$ o’rnigaqo’yishda A={1,2,3,...,n}

to’plamningixtiyoriy i, j elementlaridantuzilganjuftlikuchun $i - j$ va $\varphi(i) - \varphi(j)$ ayirmalarbirxilishoragaegabo’lsa, bujuftlikto’g’ri, birxilishoragaegabo’lmasato’g’riemasyokiinversiyatashkiletadideyiladi.

7.2-misol. $\varepsilon = \begin{pmatrix} 1 & 2 & \cdots & i & \cdots & n \\ 1 & 2 & \cdots & i & \cdots & n \end{pmatrix}$ o’rnigaqo’yishdainversiyalaryo’q.

$\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ o’rnigaqo’yishda $\{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$ juftliklarinversiyatashkiletadi.

7.8-ta’rif. $\varphi = \begin{pmatrix} 1 & 2 & \ldots & n \\ \varphi(1) & \varphi(2) & \ldots & \varphi(n) \end{pmatrix}$ o’rnigaqo’yishdainversiyalarsonijuft (toq) bo’lsa, o’rnigaqo’yishjuft (toq) o’rnigaqo’yishdeyiladi.

7.2-misolda keltirilgan $\varepsilon = \begin{pmatrix} 1 & 2 & \cdots & i & \cdots & n \\ 1 & 2 & \cdots & i & \cdots & n \end{pmatrix}$ va $\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ o’rnigaqo’yishlarjufto’rnigaqo’yishbo’ladi.

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7.9-ta'rif. $\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$ o'rnigaqo'yishdashunday i, j elementlarmavjudbo'lib, ularuchun $\varphi(i) = j, \varphi(j) = i, \varphi(s) = s, s \in A \setminus \{i, j\}$ shartlarbajarilsa, bundayo'rnigaqo'yishtranspozitsiyadeyiladi.

7.2-teorema. Harqandaytranspozitsiyatoqo'rnigaqo'yishbo'ladi.

Isbot. $\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$ o'rnigaqo'yish i ni j $i \neq j$ ga

o'tkazuvchi $\varphi(i) = j, \varphi(j) = i, \varphi(s) = s, s \in A \setminus \{i, j\}$ shartlarniqanoatlantiruvchitranspozitsiyabo'lsin. Agar

1) $i < j$ bo'lsa, $\{s, t\} \in A$ juftlikningkamidabittasi i yoki j gatengbo'lishidan, berilgano'rnigaqo'yishdainversiyamavjudligikelibchiqadi.

2) $i < s$ yoki $j < s$ bo'lsa, u holda $\{s, i\}$, $\{j, s\}$ juftliklardainversiyalaryo'q.

3) $i < s \leq j$ bo'lsa, $\{i, s\}$ juftliklardan $\{i, i+1\}$, . . . , $\{i, j\}$ lar, ya'ni $j - i$ tainversiyamavjud.

4) $i < s < j$ bo'lsa, $\{s, j\}$ lardan $\{i+1, j\}, \dots, \{j-1, j\}$ lar, ya'ni $j - i - 1$ tainversiyamavjud.

Demak, berilgantranspozitsiya $(j - i) + (j - i - 1) = 2(j - i) - 1$ tainversiyagaega, ya'nitoqo'rnigaqo'yish.

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7.10-ta'rif. $\varphi = \begin{pmatrix} 1 & 2 & \dots & n \\ \varphi(1) & \varphi(2) & \dots & \varphi(n) \end{pmatrix}$ o'rnigaqo'yishningishorasideb

$$\operatorname{sgn}\varphi = \begin{cases} 1, \text{агар} \quad \varphi - \text{жуфт}, \\ -1, \text{агар} \quad \varphi - \text{ток}. \end{cases}$$ qiymatgaaytiladi.

2.2.4. Definition. *The permutation π is called even, if* **sign** $\pi = 1$ *and π is called odd, if* **sign** $\pi = -1$. *Thus, π is even precisely when the number of inversion pairs of π is even and odd when the number of inversion pairs is odd.*

The equation **sign**$(\pi \circ \sigma) =$ **sign** π **sign** σ implies that the product of two even permutations is even, the product of two odd permutations is even, and that the product of an even and an odd permutation is odd.

7.3-teorema.O'rnigaqo'yishlarko'paytmasiningishorasi, o'rnigaqo'yishlarishoralariko'paytmasigateng.

7.4-teorema.O'rnigaqo'yishlarishorasiquyidagixossalargaega:

1) sgn funksiyamultiplikativ, ya'niharqanday $\varphi, \psi \in S_n$ laruchun $\operatorname{sgn}(\varphi\psi) = \operatorname{sgn}\varphi \cdot \operatorname{sgn}\psi$ o'rinli;

2) transpozitsiyaishorasi (-1) gateng;

3) o'zaroteskario'rnigaqo'yishlarishorasibirxil;

4) agar τ -transpozitsiyava φ ixtiyoriyo'rnigaqo'yishbo'lsa, u holda $\operatorname{sgn}(\tau\varphi) = \operatorname{sgn}(\varphi\tau) = -\operatorname{sgn}\varphi$ bo'ladi.

7.5-teorema. Harqandayikkitajuftyokitoqo'rnigaqo'yishlarko'paytmasijufto'rnigaqo'yishbo'ladi;

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Birijuftikkinchisitoqo’rnigaqo’yishlarko’paytmasitoqo’rnigaqo’yishbo’ladi.

Takrorlashuchunsavollar:

1. n-darajalio’rnigaqo’yishgata’rifbering.
2. O’rnigaqo’yishlargruppatashkiletishinitekshiring.
3. n-darajalisimmetrikgruppagamisolkeltiring.
4. Inversiyagata’rifbering.
5. Juft, toqo’rnigaqo’yishlarnita’riflang.
6. Transpozitsiyanima?
7. O’rnigaqo’yishningishorasiqandayaniqlanadi?

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